Optimal hierarchical system of a grid road network

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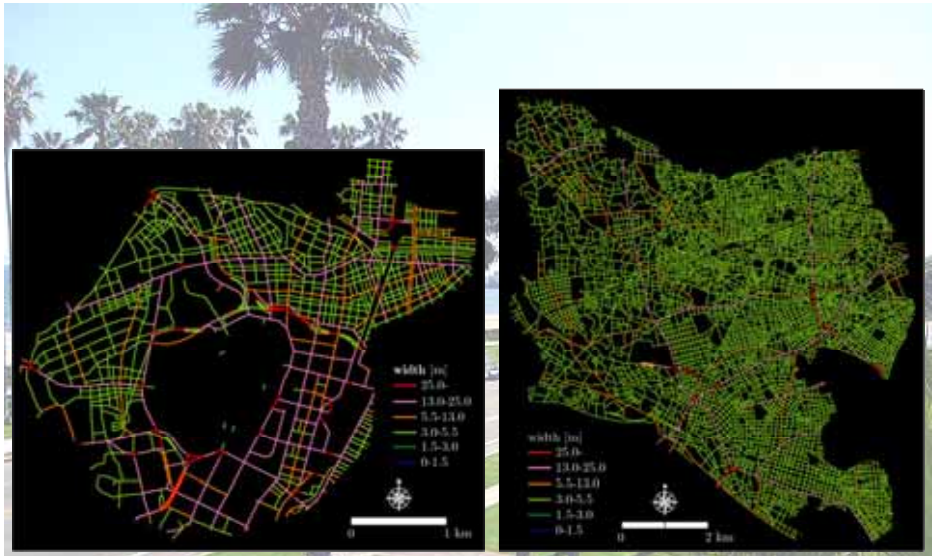
For details…

Miyagawa, M.: Optimal hierarchical system of a grid road network, *Annals of Operations Research*, 172, 349-361, 2009.

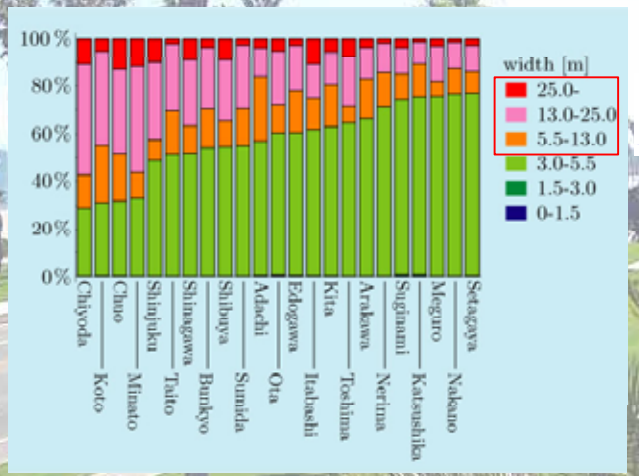


Chiyoda 71.1%

Area of major arterial roads

Setagaya 23.0%

Proportion of road areas in Tokyo



Major arterial

Proportions differ considerably among wards

Hierarchical system of road networks

Major arterial roads

Access to roadside facilities is restricted

Access roads

Traffic volume and travel speed are strictly regulated

For efficient road networks, an appropriate hierarchical system must be established

Two main approaches

Empirical		Analytical
Discrete network	Model	Continuous plane
Developing algorithms	Focus	Finding relationships
Detailed data	Travel demand	Approximated functions
Current (1986) Pirkul et al. (1991) Chopra & Tsai (2002)	Literature	Creighton et al. (1960) Fawaz & Newel (1976) Aldaihani et al. (2004)

Purpose

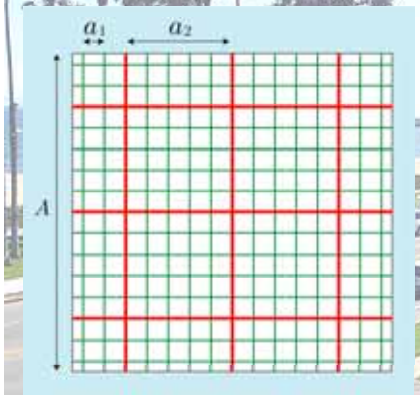
To find the hierarchical system that minimizes travel time

Ratio of road areas
major/minor arterial roads

Contents

1. Grid network model
2. Average travel time
3. Maximum travel time
4. Road network of Tokyo

Grid network model



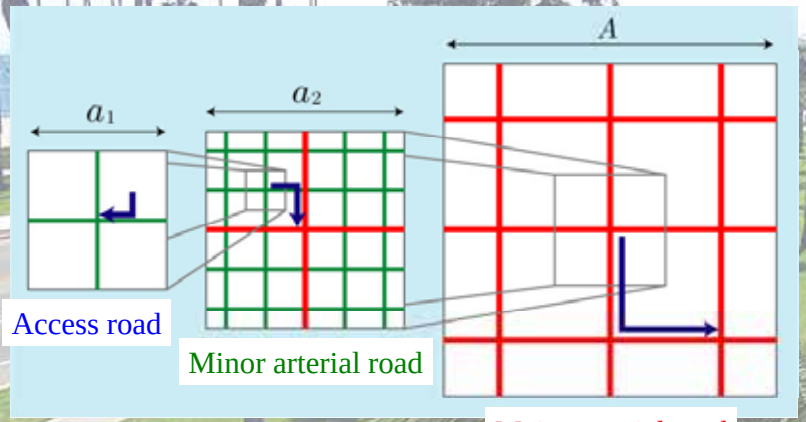
road	width	length	area
access	–	∞	–
minor arterial	w_1	Λ_1	S_1
major arterial	w_2	Λ_2	S_2

Origins and destinations are uniformly and independently distributed within the city

Find S_2/S_1 that minimizes travel time

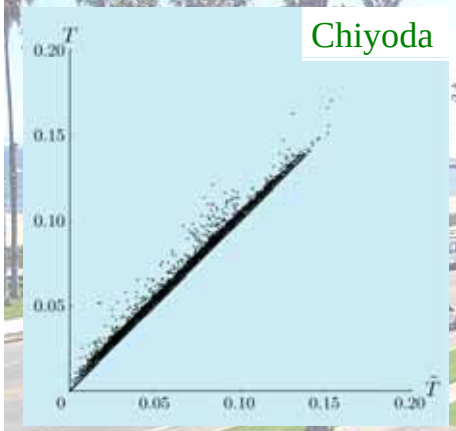
Nearest intersection routing

Every traveller uses both minor and major arterial roads

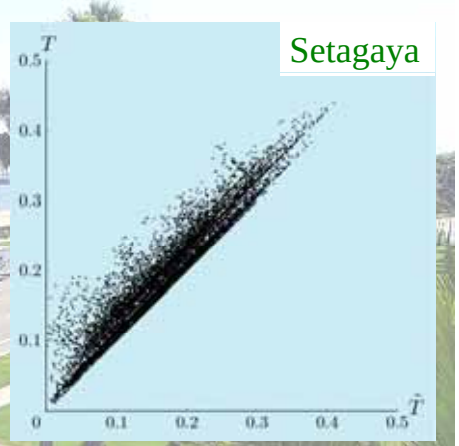


Travel time on road networks

Chiyoda

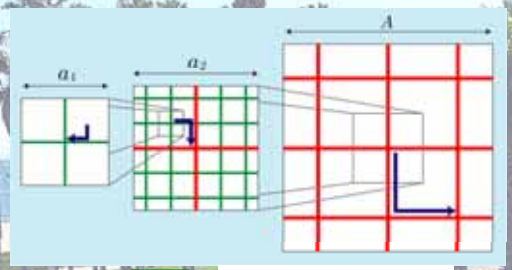


Setagaya

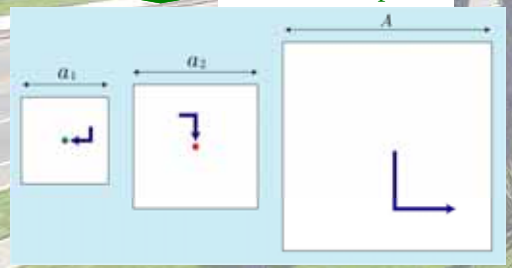


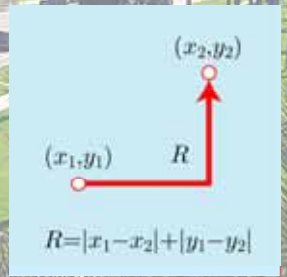
Nearest intersection routing is a good approximation of minimum travel time routing

Average travel time



Continuous plane

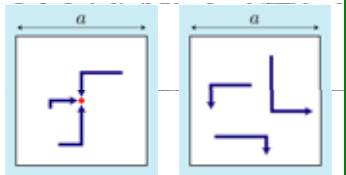




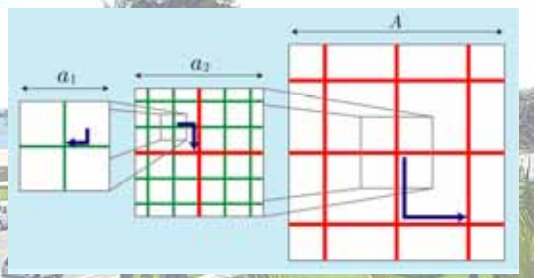
Rectilinear distance

Average travel time

Rectilinear distance in a square



$R_a = \frac{a}{2}$ $R_b = \frac{2}{3}a$



$T_0 = \frac{a_1}{2v_0}$, $T_1 = \frac{a_2}{2v_1}$, $T_2 = \frac{2A}{3v_2}$

$T = 2T_0 + 2T_1 + T_2$

Optimal ratio of road areas

$$\begin{aligned} \text{min.} \quad & T = \frac{2A^2}{v_0\Lambda_1} + \frac{2A^2}{v_1\Lambda_2} + \frac{2A}{3v_2} \\ \text{s.t.} \quad & w_1\Lambda_1 + w_2\Lambda_2 \le S \\ & \Lambda_1 \ge \Lambda_2 \ge 0. \end{aligned}$$

Total road area constraint

$$\frac{S_2^*}{S_1^*} = \frac{w_2\Lambda_2^*}{w_1\Lambda_1^*} = \sqrt{\frac{v_0 w_2}{v_1 w_1}}$$

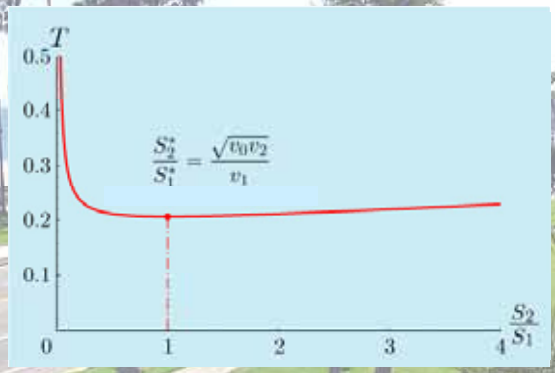
Travel speeds & road width

$$v_i = cw_i, \quad (i = 1, 2)$$

$$\frac{S_2^*}{S_1^*} = \frac{\sqrt{v_0 v_2}}{v_1}$$

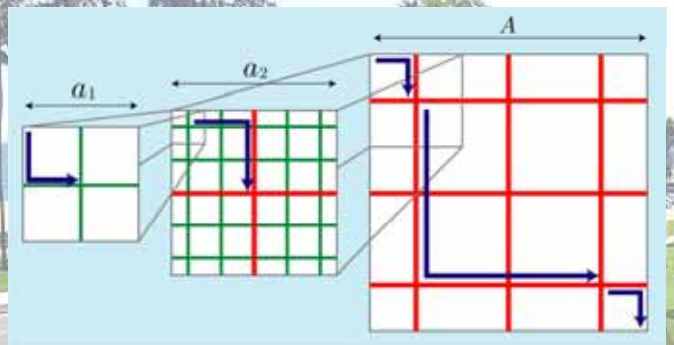
The higher the travel speed on major arterial roads v_2 becomes, the more major roads are constructed

Optimal ratio of road areas



Average travel time is almost constant for $S_2/S_1 > S_2^*/S_1^*$

Maximum travel time



$$T_0 = \frac{a_1}{v_0}, \quad T_1 = \frac{a_2 - a_1}{v_1}, \quad T_2 = \frac{2(A - a_2)}{v_2}$$

$$T_{\max} = 2T_0 + 2T_1 + T_2$$

Optimal ratio of road areas

$$\begin{aligned} \text{min.} \quad & T_{\max} = \frac{4A^2}{v_0\Lambda_1} + \frac{4A^2}{v_1}\left(\frac{1}{\Lambda_2} - \frac{1}{\Lambda_1}\right) + \frac{4A^2}{v_2}\left(\frac{1}{2A} - \frac{1}{\Lambda_2}\right) \\ \text{s.t.} \quad & w_1\Lambda_1 + w_2\Lambda_2 \le S \\ & \Lambda_1 \ge \Lambda_2 \ge 0. \end{aligned}$$

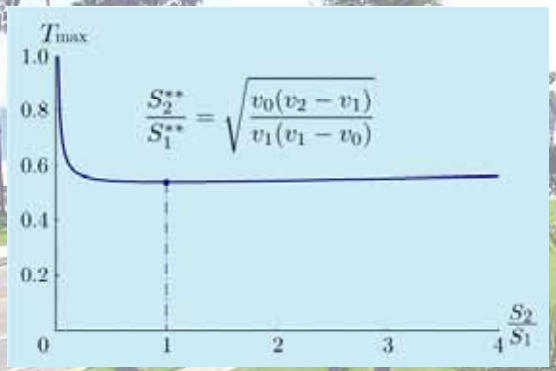
$$\frac{S_2^{**}}{S_1^{**}} = \frac{w_2\Lambda_2^{**}}{w_1\Lambda_1^{**}} = \sqrt{\frac{v_0(v_2 - v_1)w_2}{v_2(v_1 - v_0)w_1}}$$

Travel speeds & road widths

$$v_i = cw_i, \quad (i = 1, 2)$$

$$\frac{S_2^{**}}{S_1^{**}} = \sqrt{\frac{v_0(v_2 - v_1)}{v_1(v_1 - v_0)}}$$

Optimal ratio of road areas



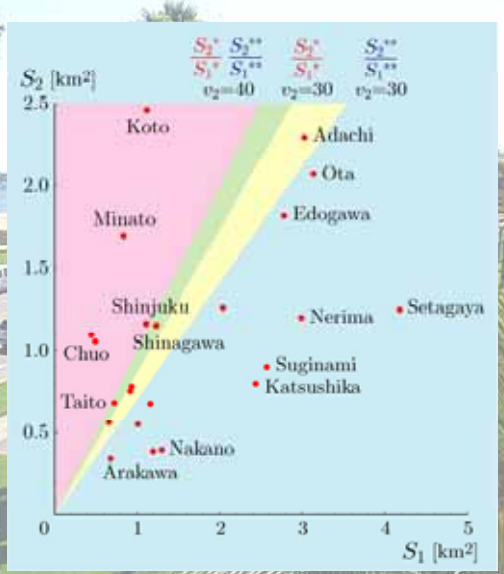
Maximum travel time is almost constant for $S_2/S_1 > S_2^{**}/S_1^{**}$

Area of minor/major arterial roads

Optimal

$S_2/S_1 > S_2^{*}/S_1^{*}$

$S_2/S_1 > S_2^{**}/S_1^{**}$



Conclusion

1. There exist optimal ratios of road areas that minimize the average and maximum travel time
2. The road networks of the central part of Tokyo are optimal